AP Calculus Summer Work - Algebra/Trig Review Name: _____

Cover Page - Please read.

Going into AP calculus, there are certain skills that have been taught to you over the previous years that I assume you have. If you do not have these skills, you will find that you will consistently get problems incorrect next year, even though you understand the calculus concepts. It is frustrating for students when they are tripped up by the algebra and not the calculus. This summer packet is intended for you to brush up and possibly relearn these topics. I assume that you have basic skills in algebra. Being able to solve equations, work with algebraic expressions, and basic factoring, for example should now be a part of you. If not, you would not be going onto AP calculus. So, only the topics I see that students consistently do not have down in their basic skill set are included here. These are skills that are used continually in A.P. Calculus. **Please write the answers to each problem in the space provided in this packet.** You may include your work on this paper or on separate pieces of papers. If you are including your work on this paper, please be neat and box your final answers, so I don't have to hunt them down!

I have suggested several websites that have full instructions on techniques necessary to solve the problems. If and when you are unsure of how to attempt these problems, checkout these websites. Don't fake your way through these problems. As stated, students are notoriously weak in them, even students who have achieved well prior to AP Calculus. Use the websites!! Realize also that certain concepts are interrelated. Domain, for example, may require you to be expert at working with inequalities. Solving quadratic equations may involve techniques used in solving fractional equations. This packet is **due the first day back in school** in the fall. You will be uploading these pages onto Canvas. **It will be graded**. You need to get off to a good start so spend some quality time on this packet this summer. Work needs to be shown when needed. Also do not rely on the calculator. Half of your AP exam next year is taken without the calculator. So paper and pencil techniques only. I want these techniques to be relatively fresh in your mind in the fall, so begin working on this packet about four weeks before the start of school. Do not wait to do them at the very last minute. These take time. If you have questions about any of these problems or techniques used in solving them, contact me at rmooty@menaulschool.org

Here is a good site for most algebra topics: http://www.purplemath.com/modules/index.htm
You can also google lessons by Brian McLogan (PreCalculus) and Professor Leonard (PreCalculus and Calculus)

Beginning algebra topics:
1. Exponents and Radicals; Negative and fractional exponents

Intermediate algebra topics
2. Domain
3. Solving inequalities: absolute value
4. Solving inequalities: quadratic
5. Special factoring formulas
6. Function transformation
7. Factor theorem (p over q method)
8. Even and odd functions
9. Solving quadratic equations and quadratic formula

Advanced algebra topics
10. Asymptotes
11. Complex fractions
12. Composition of functions
13. Solving rational (fractional) equations
14. Log properties
15. Exponential and log equations

16. Expanding and simplifying log expressions

Trigonometry
http://www.mathematicshelpcentral.com/index.html

Once in the site, go to lecture notes.
17. The Unit Circle and Identities/Formulas
18. Basic right-angle trig
19. Trig equations
20. Independent investigations and pra

Topic1 - Exponents and Radicals

1. Write the radical expression using exponents.

(a) $\frac{1}{\sqrt{13}}$ (b) $\sqrt[6]{3x^2y^5}$

2. Write the exponential expression using radicals.

(a) $10^{-\frac{9}{2}}$ (b) $a^{\frac{7}{8}}$

3. Evaluate the expression without using a calculator.

(a) $\frac{3}{3^{-2}}$ (b) $\sqrt[4]{256}$

(c) $\sqrt[3]{150}\sqrt[3]{180}$ (d) $\sqrt{72}+\sqrt{50}$

4. Simplify the expression. Assume the variables denote any real numbers.

(a) $\sqrt[4]{\sqrt{65,536x^8}}$ (b) $\left(9x^8y^4\right)\left(\frac{1}{3}x^4y^6\right)$ (c) $\frac{\left(x^9y^6\right)^2\left(x^2y^7\right)^{-4}}{x^8y^{12}}$

5. Simplify the expression. Write your answer with positive exponents only. Assume all variables are positive numbers.

(a) $x^{\frac{1}{3}}x^{\frac{1}{7}}$ (b) $(4b)^{\frac{1}{2}}\left(7b^{\frac{2}{3}}\right)$ (c) $\left(\frac{-2x^{\frac{1}{5}}}{y^{\frac{1}{2}}z^{\frac{1}{3}}}\right)^4$

(d) $\left(x^{-5}y^{8}z^{4}\right)^{-\frac{10}{3}}$ (e) $\left(\frac{a^{2}b^{-3}}{x^{-3}y^{2}}\right)^{3}\left(\frac{x^{-4}b^{-2}}{a^{\frac{3}{2}}y^{\frac{1}{3}}}\right)$

6. Simplify. Write using positive exponents only.

a) $-5\left(\frac{3}{2}\right)(4-9x)^{-\frac{1}{2}}(-9)$ (b) $2\left(\frac{2}{2-x}\right)\left[\frac{-2}{(2-x)^{2}}\right]$

(c) $-\frac{x^{-\frac{1}{2}}}{2}\sin\sqrt{x}$ (d) $\frac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^{3}}}$

(e) $-4\left(\frac{2x-1}{2x+1}\right)^{-3}\left[\frac{2(2x+1)-2(2x-1)}{(2x+1)^{2}}\right]$ (f) $\frac{\frac{1}{2}(2x+5)^{-\frac{3}{2}}}{\frac{3}{2}}$

(g) $\left(\frac{1}{x^{-2}}+\frac{4}{x^{-1}y^{-1}}+\frac{1}{y^{-2}}\right)^{-\frac{1}{2}}$

Topic 2 - Domain

Find the domain of each function. Write your answer in interval notation, if possible.

1. $y = \dfrac{3x-2}{4x+1}$

2. $y = \dfrac{x^2-4}{2x+4}$

3. $y = \dfrac{x^2-5x-6}{x^2-3x-18}$

4. $y = \dfrac{2^{2-x}}{x}$

5. $y = \sqrt{x-3} - \sqrt{x+3}$

6. $y = \dfrac{\sqrt{2x-9}}{2x+9}$

7. $y = \dfrac{x^2+8x+12}{\sqrt[4]{x+5}}$

8. $y = \sqrt{x^2-5x-14}$

9. $y = \dfrac{\sqrt[3]{x-6}}{\sqrt{x^2-x-30}}$

10. $y = \log(2x-12)$

11. $y = \sqrt{\tan x}$

12. $y = \dfrac{x}{\cos x}$

Topic 3 - Solving Inequalities (Absolute Value)

Write each absolute value function as a piecewise function in this format:

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases} = \begin{cases} \_\_\_\_\_ & \text{if } x \geq c \\ \_\_\_\_\_ & \text{if } x < c \end{cases}$$

1. $y = |2x - 4|$

2. $y = |6 + 2x| + 1$

3. $y = |4x + 1| + 2x - 3$

Solve the following absolute value inequalities. Write your answers in interval notation.

4. $|x - 3| > 12$

5. $|x - 3| \leq 4$

6. $|10x + 8| > 2$

7. $|3x - 4| > -2$

8. $|x - 6| < -2$

9. $|x + 1| \leq |x - 3|$

Topic 4 - Solving Inequalities (Quadratics)

Write each absolute value expressions as piecewise expressions with simplified domain (see topic 3).

1. $\left|x^2-1\right|$ 2. $\left|x^2+x-12\right|$ 3. $\left|x^2+4x+4\right|$

Solve the following inequalities by factoring and making the appropriate sign chart. Write your solution in exact form, if possible, and in interval notation.

4. $x^2-16>0$ 5. $x^2+6x-16>0$ 6. $x^2-3x\geq 10$

7. $2x^2+4x\leq 3$ 8. $x^3+4x^2-x\geq 4$ 9. $2\sin^2 x\geq \sin x;\ [0,2\pi)$

Topic 5 - Special Factorization

Factor completely

1. x^3+8

2. x^3-8

3. $27x^3-125y^3$

4. x^4+11x^2-80

5. $ac+cd-ab-bd$

6. $2x^2+50y^2-20xy$

7. $x^2+12x+36-9y^2$

8. $x^3-xy^2+x^2y-y^3$

9. $(x-3)^2(2x+1)^3+(x-3)^3(2x+1)^2$

Topic 6 - Function Transformation

If $f(x) = x^2 - 1$, describe in words what the following would do to the graph of $f(x)$.

1. $f(x) - 4$

2. $f(x - 4)$

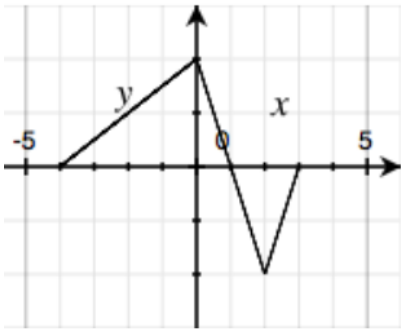
3. $-f(x + 2)$

4. $5f(x) + 3$

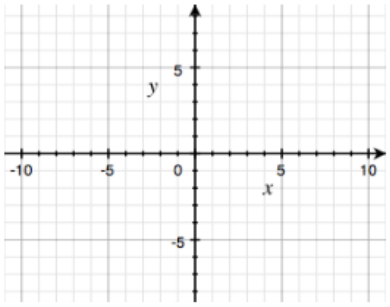
5. $f(2x)$

6. $|f(x)|$

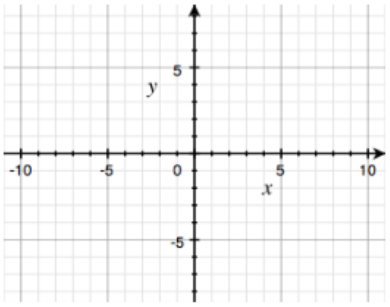
Here is a graph of $y = f(x)$. Sketch the following graphs.



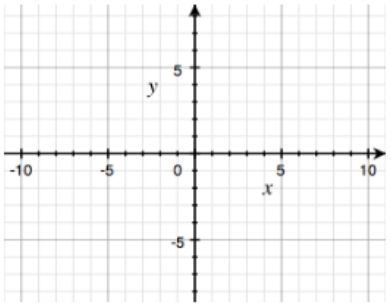
7. $y = 2f(x)$



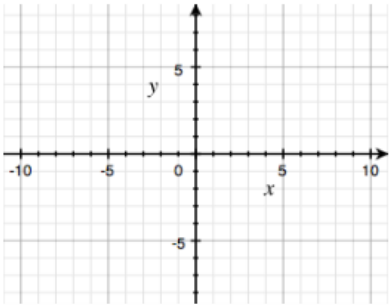
8. $y = -f(x)$



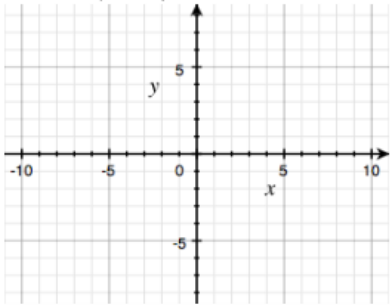
9. $y = f(x - 1)$



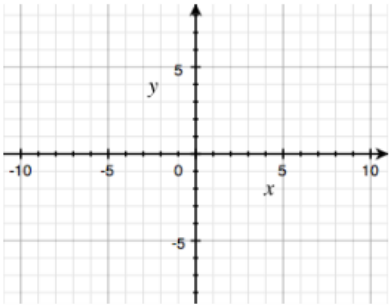
10. $y = f(x + 2)$



11. $y = |f(x)|$



12. $y = f|x|$



Topic 7 - Graphs of Polynomials and the Factor Theorem

For each polynomial function below, without using a graphing utility, state the possible number of turning points, the y-intercept and end behavior of the graph. Sketch the graph of the function based on your description.

1. $P(x) = x^3 + 4x^2 + x - 6$

2. $P(x) = -x^3 - 5x^2 + 2x + 24$

3. $P(x) = -2x^2 + 19x + 20$

4. $P(x) = x^4 + 5x^3 + 6x^2 - 4x - 8$

Write each polynomial, $P(x)$ in factored form. Then solve $P(x) = 0$ (Hint: $\frac{p}{q}$ and synthetic division)

5. $P(x) = x^3 + 4x^2 + x - 6$

6. $P(x) = x^3 + 5x^2 - 2x - 24$

7. $P(x) = x^3 + 2x^2 - 19x - 20$

8. $P(x) = x^4 + 5x^3 + 6x^2 - 4x - 8$

Topic 8 - Even and Odd Functions

A function is even if $f(x) = f(-x)$. A function is odd if $-f(x) = f(-x)$

Show work to determine if the relation is even, odd or neither. Check the graph of each relation using Desmos. Write your observation of graphs of odd and even relations.

1. $f(x) = 2x^2 - 7$

2. $f(x) = -4x^3 - 2x$

3. $f(x) = 4x^2 - 4x + 4$

4. $f(x) = x - \frac{1}{x}$

5. $f(x) = |x| - x^2 + 1$

6. $5x^2 - 6y = 1$

7. $y = e^x - \frac{1}{e^x}$

8. $3y^3 = 4x^3 + 1$

9. $3x = |y|$

Topic 9 - Solving Quadratic and Quadratic-Type Equations

Give simplest, exact solutions to the following equations.

1. $7x^2 - 3x = 0$

2. $4x(x-2) - 5x(x-1) = 2$

3. $x^2 + 6x + 4 = 0$

4. $2x^2 - 3x + 3 = 0$

5. $2x^2 - (x+2)(x-3) = 12$

6. $x + \frac{1}{x} = \frac{13}{6}$

7. $x^4 - 9x^2 + 8 = 0$

8. $x - 10\sqrt{x} + 9 = 0$

9. $\frac{1}{x^2} - \frac{1}{x} = 6$

Topic 10 - Asymptotes

For each function, state any holes, vertical and horizontal asymptotes (if they exist).

1. $y = \frac{x}{x-3}$

2. $y = \frac{x+4}{x^2-1}$

3. $y = \frac{x+4}{x^2+1}$

4. $y = \frac{x^2-2x+1}{x^2-3x-4}$

5. $y = \frac{x^2-9}{x^3+3x^2-18x}$

6. $y = \frac{2x^2+6x}{x^3-3x^2-4x}$

7. $y = \frac{x^2-x-6}{x^3-x^2+x-6}$

8. $y = \frac{2x^3}{x^3-1}$

9. $y = \frac{\sqrt{x}}{2x^2-10}$

Topic 11 - Complex Fractions

Simplify.

1. $\dfrac{x}{x-\frac{1}{2}}$

2. $\dfrac{\frac{1}{x}+4}{\frac{1}{x}-2}$

3. $\dfrac{x-\frac{1}{x}}{x+\frac{1}{x}}$

4. $\dfrac{\frac{3}{x}-\frac{4}{y}}{\frac{4}{x}-\frac{3}{y}}$

5. $\dfrac{1-\frac{2}{3x}}{x-\frac{4}{9x}}$

6. $\dfrac{\frac{x^2-y^2}{xy}}{\frac{x+y}{y}}$

7. $\dfrac{x^{-3}-x}{x^{-2}-1}$

8. $\dfrac{\frac{x}{1-x}+\frac{1+x}{x}}{\frac{1-x}{x}+\frac{x}{1+x}}$

9. $\dfrac{\frac{4}{x-5}+\frac{2}{x+2}}{\frac{2x}{x^2-3x-10}+3}$

Topic 12- Composition of Functions

If $f(x) = x^2$, $g(x) = 2x - 1$ and $h(x) = 2^x$, find the following.

1. $f\big(g(2)\big)$

2. $g\big(f(2)\big)$

3. $f\big(h(-1)\big)$

4. $h\big(f(-1)\big)$

5. $g\left(f\left(h\left(\frac{1}{2}\right)\right)\right)$

6. $\big(f \circ g\big)(x)$

7. $\big(g \circ f\big)(x)$

8. $\big(g \circ g\big)(x)$

9. $f\big(h(x)\big)$

Topic 13 - Rational Equations

State the excluded values for each equation. Solve for x.

1. $\frac{2}{3}-\frac{5}{6}=\frac{1}{x}$

2. $x+\frac{6}{x}=5$

3. $\frac{x+1}{3}-\frac{x-1}{2}=1$

4. $\frac{x-5}{x+1}=\frac{3}{5}$

5. $\frac{60}{x}-\frac{60}{x-5}=\frac{2}{x}$

6. $\frac{2}{x+5}+\frac{1}{x-5}=\frac{16}{x^2-25}$

7. $\frac{x}{x-2}+\frac{2x}{4-x^2}=\frac{5}{x+2}$

8. $\frac{x}{2x-6}-\frac{3}{x^2-6x+9}=\frac{x-2}{3x-9}$

9. $\frac{2x+3}{x-1}=\frac{10}{x^2-1}+\frac{2x-3}{x+1}$

Topic 14 - Properties of Logarithms and Exponents

Evaluate the expression **without a calculator**

1. $\log_3 6 - \log_3 8 + \log_3 108$

2. $\log_4 64^{600}$

3. $\ln\left(\ln e^{e^{600}}\right)$

4. $\log_7 343$

5. $10^{\log \pi}$

6. $e^{\ln 5}$

7. $\log_3 189 - \log_3 7$

8. $\log_5 \sqrt[5]{625}$

9. $(\log_3 7)(\log_7 13)$

Use the Change of Base Formula and a calculator to evaluate the logarithm, correct to six decimal places. Use either natural or common logarithms.

10. $\log_7 2.63$

11. $\log_7 539$

12. $\log_5 e^5$

Topic 15 - Logarithmic and Exponential Equations

Express the equation in logarithmic form.

1. $8^x = m$ 2. $e^x = 5$ 3. $x^{-2} = e$

Write the logarithmic equation in its equivalent exponential form, then solve for *x*.

4. $\ln x = 3$ 5. $\log_x 81 = -4$ 6. $\log_x\left(\frac{1}{64}\right) = 3$

Find the solution of the exponential equations in exact form and correct to four decimal places.

7. $e^{2-3x} = 12$ 8. $3^{5x-2} = 4$ 9. $5\left(1 + 10^{6x}\right) = 12$

10. $5^{5x+1} = 2^{x-3}$ 11. $\frac{27}{1+e^{-x}} = 2$ 12. $e^{4x} + 2e^{2x} - 8 = 0$

Solve the logarithmic equation for *x*. Give exact solution and correct to four decimal places.

13. $\ln(1+x) = 1$ 14. $2 - \ln(7-x) = 0$ 15. $\ln(x-1) + \ln(x+2) = 1$

Topic 16 - Expanding and simplifying log expressions

Use the Laws of Logarithms to rewrite the expression below in a form with no logarithm of a product, quotient, or power.

1. $\log_4\left(x(x-9)\right)$

2. $\log_a\left(\dfrac{x^2}{yz^7}\right)$

3. $\ln\left(x\cdot\sqrt[9]{\dfrac{y}{z}}\right)$

4. $\ln\sqrt{\dfrac{x^2+6}{\left(x^2+1\right)\left(x^5-4\right)^2}}$

5. $\log\sqrt[6]{x\sqrt[6]{y\sqrt[6]{z}}}$

6. $\ln\left(\dfrac{z^7\cdot\sqrt{x}}{\sqrt[5]{y^2+4y+15}}\right)$

Use the laws of logs to rewrite the expression as a single logarithm.

7. $\log_3 2+2\log_3 2$

8. $\ln 14+\frac{1}{2}\ln 3-\ln 2$

9. $\ln 6+5\ln x+9\ln\left(x^2+9\right)$

10. $5\log x-\frac{1}{3}\log\left(x^2+1\right)+4\log(x-1)$

11. $\ln(x+y)+\ln(x-y)-2\ln z$

12. $2\left(\log_3 x+2\log_3 y-4\log_3 z\right)$

Topic 17 - The Unit Circle and Identities/Formulas

Fill in the cells with exact values.

θ	$\cos\theta$	$\sin\theta$	$\tan\theta$
0			
$\frac{\pi}{6}$			
$\frac{\pi}{4}$			
$\frac{\pi}{3}$			
$\frac{\pi}{2}$			
π			
$\frac{7\pi}{6}$			
$-\frac{3\pi}{4}$			
$\frac{16\pi}{3}$			
$-\frac{23\pi}{6}$			

State the identities from memory. (Yes, I do realize you can cheat here, but the idea is that you have these memorized because you will not be allowed to use the identities sheet on any exams in the Calculus class)

Pythagorean Identities:

Quotient Identities:

Reciprocal Identities:

Odd/Even Identities:

Sum and Difference Formulas:

Double Angle Formulas:

Half-Angle Formulas:

Topic 18 - Basic Right-Angle Trigonometry

If point P is on the terminal side of θ, find all six trigonometric functions of θ. Draw a triangle in the appropriate quadrant.

1. $P(-2,4)$

2. $P\left(\sqrt{5},-2\right)$

3. If $\cos\theta = -\frac{5}{13}$, θ in quadrant II
find $\sin\theta$ and $\tan\theta$

4. If $\cot\theta = 3$, θ in quadrant III
find $\sin\theta$ and $\cos\theta$

Evaluate **without using a calculator.**

5. $\sin^2 225° - \cos^2 300°$

6. $\left(6\sec\pi - 4\cot\frac{\pi}{2}\right)^2$

7. $\left(4\cos\frac{\pi}{6} - 6\sin\frac{2\pi}{3}\right)^{-2}$

Find the missing side-lengths and angles, correct to three decimal places,

8.
$A = \_\_\_°$ $a = 21.7$
$B = 16°$ $b = \_\_\_$
$C = 90°$ $c = \_\_\_$

9.
$A = \_\_\_°$ $a = 6$ feet
$B = \_\_\_°$ $b = \_\_\_\_$ inches
$C = 90°$ $c = 95$ inches

Topic 19 - Solving Trig Equations

Solve each equation on the interval $[0, 2\pi)$

1. $\sin x = \frac{1}{2}$

2. $\cos^2 x = \cos x$

3. $2\cos x + \sqrt{3} = 0$

4. $4\sin^2 x = 1$

5. $2\sin^2 x + \sin x = 1$

6. $\cos^2 x + 2\cos x = 3$

7. $2\sin x \cos x + \sin x = 0$

8. $8\cos^2 x - 2\cos x = 1$

9. $\sin^2 x - \cos^2 x = 0$

Topic 20 - Trigonometric Formulas and Identities

1. **Rewrite the expression as a trigonometric function of an angle measure.**

a) $\cos 5\theta\cos\theta + \sin 5\theta\sin\theta$ b) $\dfrac{\tan 4\theta + \tan 2\theta}{1 - \tan 4\theta\tan 2\theta}$ c) $5 \sin 3\theta\cos 3\theta$

2. Given $\sin\theta = \dfrac{12}{37}$ and $\dfrac{\pi}{2} \le \theta \le \pi$, find the exact value of $\cos\dfrac{\theta}{2}$.

3. Solve $\cos 2\theta = 2\sin^2\theta$ for $0 \le \theta < 2\pi$.

4. Verify the following identities algebraically.

1. $\dfrac{\sin^2\theta}{1 - \cos\theta} = \dfrac{1 + \sec\theta}{\sec\theta}$

2. $\tan\theta + \cot\theta = \dfrac{1}{\sin\theta\cos\theta}$

3. $\dfrac{\sec\theta}{\csc\theta - \cot\theta} - \dfrac{\sec\theta}{\csc\theta + \cot\theta} = 2\csc\theta$

4. $\dfrac{1}{1 + \sin\theta} + \dfrac{1}{1 - \sin\theta} = 2\sec^2\theta$

5. $\sin^4 x = \dfrac{3}{8} - \dfrac{1}{2}\cos 2x + \dfrac{1}{8}\cos 4x$

Independent Review/Investigations

Use this link. Follow the instructions to create an account. Just work the first 4 weeks. Mr. Trowbridge says this is an excellent algebra review for Calculus.

Course | Pre-University Calculus | edX

(Go to Edx, then search for Pre-University Calculus)

Please scan your work into one file. Be ready to upload that file onto Canvas the first day of school!